

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Superstring-inspired supergravity as the universal source of inflation and quintessence [☆]

S. James Gates Jr., Sergei V. Ketov ^{*}

Physics Department, University of Maryland, College Park, MD 20742, USA

ARTICLE INFO

Article history:

Received 5 February 2009

Accepted 4 March 2009

Available online 5 March 2009

Editor: T. Yanagida

ABSTRACT

We prove (in superspace) the equivalence between the higher-derivative $N = 1$ supergravity, defined by a holomorphic function F of the chiral scalar curvature superfield, and the standard theory of a chiral scalar superfield with a chiral superpotential W , coupled to the (minimal) Poincaré supergravity in four spacetime dimensions. The relation between the holomorphic functions F and W is found. It can be used as the technical framework for the possible scenario unifying the early Universe inflation and the present Universe acceleration. We speculate on the possible origin of our model as the effective supergravity generated by quantum superstrings, with a dilaton–axion field as the leading field component of the chiral superfield.

© 2009 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).

1. Introduction

Inflation (i.e. a phase of ‘rapid’ accelerated expansion) in the early Universe [1] nicely predicts the homogeneity of our Universe at large scales, its large size and entropy, as well as an almost scale-invariant spectrum of cosmological perturbations, in good agreement with the high precision CMB measurements [2]. A mechanism (and details) of inflation is usually based on a ‘slow-roll’ scalar field (inflaton) with a proper scalar potential [1]. It follows from astronomical observations of Supernova Ia [3] that the present Universe is accelerating due to the mysterious ‘dark energy’ which violates the strong energy condition in General Relativity. Dark energy is also needed to prevent a formation of super-large clusters of galaxies [4], so it begs for a theoretical proof of its existence or an alternative explanation of the present acceleration from a fundamental theory of gravity. The most naive explanation of the dark energy by a cosmological constant is not satisfactory because of its time-independence and enormous fine-tuning. A better model is provided by a scalar field (quintessence) whose scalar potential is tuned ‘by hand’ [5]. Hence, the true theoretical challenge is to explain the origin of inflaton and quintessence,

as well as provide theoretical tools for a derivation of the scalar potential.

The expected scale of inflation is close to that of Grand Unification [1], so the inflation may be due to some Planck scale physics or quantum gravity. A consistent and universal approach to quantum gravity and very high-energy particles physics is available due to theory of superstrings (or M-theory) [6]. Due to its putative fundamental nature, string theory is expected to be valid at all energy scales, which would make it indispensable in any effort to unify the UV gravity (in the very early Universe) with the IR gravity (in the present Universe). String theory should also explain the origin of quintessence (or dark energy). Assuming the validity of the effective field-theoretical description of string theory, it is reasonable to study both inflation and quintessence within the effective supergravity framework, because local supersymmetry is required for consistency of strings. Of course, supersymmetry is broken in the IR, e.g. spontaneously.

In this Letter we would like to propose a possible *geometrical* origin of the inflaton and quintessence, as described by a *single* scalar field in the supergravity model modified by higher-order supercurvature terms. The latter may originate from quantum (non-perturbative) superstrings, though we do not have a compelling reason for that. We also assume that (i) string theory is compactified down to four spacetime dimensions, (ii) all of its moduli are stabilized (e.g. by fluxes [7]), and (iii) local supersymmetry is broken to $N = 1$ in uncompactified four dimensions whose geometry is described by the FRLW metric with a scale factor $a(t)$ of physical time t , and $k = (-1, 0, +1)$,

$$ds_{\text{FRW}}^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]. \quad (1.1)$$

[☆] Supported in part by the endowment of the John S. Toll Professorship, the University of Maryland Center for String & Particle Theory, US National Science Foundation Grant PHY-0354401, and the Japanese Society for Promotion of Science (JSPS).

^{*} Corresponding author. Permanent address: Department of Physics, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-1397, Japan.

E-mail addresses: gates@wam.umd.edu (S.J. Gates Jr.), ketov@phys.metro-u.ac.jp (S.V. Ketov).

2. Basic facts of $N = 1$ superfield supergravity

The chiral superspace density (in the supersymmetric gauge-fixed form) reads

$$\mathcal{E}(x, \theta) = e(x) [1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x)], \quad (2.1)$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi_\alpha^a = e_\mu^a \psi_\alpha^\mu$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle Greek letters $\mu, \nu, \dots = 0, 1, 2, 3$ for curved spacetime vector indices, the lower case early Latin letters $a, b, \dots = 0, 1, 2, 3$ for flat (target) space vector indices, and the lower case early Greek letters $\alpha, \beta, \dots = 1, 2$ for chiral spinor indices.

The solution of the superspace Bianchi identities and the constraints defining the $N = 1$ Poincaré-type minimal supergravity results in only three relevant superfields \mathcal{R} , \mathcal{G}_a and $\mathcal{W}_{\alpha\beta\gamma}$ (as parts of the supertorsion), subject to the off-shell relations [8]

$$\mathcal{G}_a = \bar{\mathcal{G}}_a, \quad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}, \quad \bar{\nabla}_\alpha \mathcal{R} = \bar{\nabla}_\alpha \mathcal{W}_{\alpha\beta\gamma} = 0, \quad (2.2)$$

and

$$\bar{\nabla}_\alpha \dot{\mathcal{G}}_{\alpha\dot{\alpha}} = \nabla_\alpha \mathcal{R}, \quad \nabla^\gamma \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \nabla_\alpha \dot{\mathcal{G}}_{\beta\dot{\alpha}} + \frac{i}{2} \nabla_\beta \dot{\mathcal{G}}_{\alpha\dot{\alpha}}, \quad (2.3)$$

where $(\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}} \cdot \nabla_{\alpha\dot{\alpha}})$ represent the curved superspace $N = 1$ supercovariant derivatives, and bars denote complex conjugation.

The covariantly chiral complex scalar superfield \mathcal{R} has the scalar curvature R as the coefficient at its θ^2 term, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor, $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2}g_{\mu\nu}R$, as the coefficient at its $\theta\sigma^a\bar{\theta}$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the Weyl tensor $W_{\alpha\beta\gamma\delta}$ as the coefficient at its linear θ^δ -dependent term. Since we are interested in merely bosonic contributions, we drop the fermionic (spinor) components in what follows (except the gravitino-induced bosonic torsion).

The chiral density integration formula reads [8]

$$\int d^4x d^2\theta \mathcal{E} \mathcal{L} = \int d^4x e \{ \mathcal{L}_{\text{last}} + B \mathcal{L}_{\text{first}} \}, \quad (2.4)$$

where we have introduced the field components of the covariantly chiral superfield Lagrangian $\mathcal{L}(x, \theta)$, $\bar{\nabla}^\alpha \mathcal{L} = 0$, as follows (the vertical bars denote the leading component of a superfield):

$$\mathcal{L} = \mathcal{L}_{\text{first}}(x), \quad \nabla^2 \mathcal{L} = \mathcal{L}_{\text{last}}(x). \quad (2.5)$$

In particular, we have

$$\mathcal{R} = \frac{1}{3} \bar{B} = \frac{1}{3} (S + iP),$$

$$\nabla^2 \mathcal{R} = \frac{1}{3} \left(R - \frac{i}{2} \varepsilon^{abcd} R_{abcd} \right) + \frac{4}{9} \bar{B} B, \quad (2.6)$$

where we have kept the purely imaginary contribution $iR_{\text{tor}} \equiv \frac{i}{2} \varepsilon^{abcd} R_{abcd}$ because it does not vanish in supergravity due to the gravitino- (and matter-) induced torsion T_{abc} , with $R_{\text{tor}} \propto (\nabla T + T^2)$.

3. Proposal

A generic supergravity Lagrangian (e.g. representing the supergravitational part of the superstring effective action) in superspace is given by

$$\mathcal{L} = \mathcal{L}(\mathcal{R}, \mathcal{G}, \mathcal{W}, \dots), \quad (3.1)$$

where the dots stand for arbitrary covariant derivatives of the supergravity superfields introduced in Section 2. Since the Weyl

tensor vanishes for any scale factor in the FRLW metric (1.1), it is always consistent to take $\mathcal{W}_{\alpha\beta\gamma} = 0$ when discussing the FRLW dynamics (but not its perturbations!). Imposing further $\dot{\mathcal{G}}_{\alpha\dot{\alpha}} = 0$ would be too restrictive because of the Bianchi identities (2.3), since they would imply $R = \text{const}$. Dropping the terms with derivatives would generically be inconsistent by the same reason. Nevertheless, we would like to concentrate on the particular sector of the theory (3.1), by ignoring the vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ and all the derivatives of the superfield \mathcal{R} in Eq. (3.1). Besides having a simplification, we believe that the non-scalar arguments in the effective action are not relevant for the dynamics of the FRLW scale factor (but they are expected to be relevant e.g. for addressing the cosmological singularity [9]). So, the effective modified supergravity action we propose is given by

$$S_F = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \quad (3.2)$$

with some holomorphic function $F(\mathcal{R})$ presumably generated by strings. Besides manifest local $N = 1$ supersymmetry, the action (3.2) also possess the *auxiliary freedom* [10], since the auxiliary field B does not propagate. It distinguishes our action from other possible choices. In addition, the action (3.2) automatically gives rise to a spacetime torsion.

Most importantly, despite of the apparent presence of higher derivatives, our action (3.2) is classically equivalent to the *standard* supergravity minimally coupled to a chiral ‘matter’ superfield whose chiral superpotential is dictated by the chiral function F (see Section 8). For instance, the purely gravitational part of the action (3.2) in components is obtained by eliminating the auxiliary field $B = S - iP$ via its algebraic equation of motion. It results in the modified gravity action having the form

$$S_f = \int d^4x \sqrt{-g} f(R, R_{\text{tor}}) \quad (3.3)$$

whose function f of scalar curvature R and torsion R_{tor} is dictated by the holomorphic function F of the master action (3.2). The gravitational part of the action (3.3) can be put into the Brans–Dicke gravity form via a Legendre transform, whereas the Brans–Dicke gravity itself is well known to be equivalent to a scalar-tensor gravity [11]. Those steps also allow us to ensure the ghost-freedom of our action (3.2). When starting from superstrings, the ghost-freedom is automatic. The classical equivalence between certain higher-derivative supergravities and standard supergravity coupled to matter was observed in Ref. [12] by the use of the superconformal tensor calculus [13]. In the next sections we describe the weak coupling limit of our model and its equivalent forms in superspace.

4. Connection to General Relativity

Applying the chiral density formula (2.4) to our Eq. (3.2) yields the purely bosonic Lagrangian

$$F'(\bar{X}) \left[\frac{1}{3} R_* + 4\bar{X}X \right] + 3XF(\bar{X}) + \text{H.c.}, \quad (4.1)$$

where primes denote differentiation. We have also introduced the notation

$$X = \frac{1}{3} B, \quad R_* = R - iR_{\text{tor}}. \quad (4.2)$$

Varying Eq. (4.1) with respect to the auxiliary fields X and \bar{X} gives rise to an algebraic equation on the auxiliary fields,

$$3\bar{F} + X(4\bar{F}' + 7F') + 4\bar{X}XF'' + \frac{1}{3}F''R_* = 0, \quad (4.3)$$

and its conjugate,

$$3F + \bar{X}(4F' + 7\bar{F}') + 4\bar{X}X\bar{F}'' + \frac{1}{3}\bar{F}''\bar{R}_* = 0. \quad (4.4)$$

First, let us consider the simple special case when

$$F'' = 0, \quad \text{or, equivalently, } F(\mathcal{R}) = f_0 + f_1\mathcal{R}, \quad (4.5)$$

with some complex constants f_0 and f_1 , where $\text{Re } f_1 < 0$. Then Eq. (4.4) is easily solved as

$$\bar{X} = \frac{-3(f_0 + f_1 R_*)}{4f_1 + 7\bar{f}_1}. \quad (4.6)$$

Substituting the solution (4.6) back into the Lagrangian (4.1) yields

$$\begin{aligned} \frac{2}{3}(\text{Re } f_1)R_* - \frac{9|f_0|^2}{14(\text{Re } f_1)} &\equiv -\frac{1}{2\kappa^2}R_* - \Lambda \\ &= -\frac{1}{2\kappa^2}R(\Gamma + T) - \Lambda, \end{aligned} \quad (4.7)$$

where we have introduced the standard gravitational coupling constant $\kappa_0 = M_{\text{Planck}}^{-1}$ in terms of the (reduced) Planck mass, the standard supergravity connection (i.e. Christoffel symbols Γ plus torsion T), and the cosmological constant Λ ,

$$\kappa = \sqrt{\frac{3}{4|\text{Re } f_1|}}, \quad \Lambda = \frac{-9|f_0|^2}{14|\text{Re } f_1|}. \quad (4.8)$$

As is clear from the above equations, the cosmological constant in supergravity is always negative, as is required by local supersymmetry [8]. Since we are not interested in the standard supergravity or General Relativity here, we assume that $F'' \neq 0$ in what follows.

5. Superfield Legendre transform

Our superfield action (3.2) is classically equivalent to another action

$$S_V = \int d^4x d^2\theta \mathcal{E}[\mathcal{Z}\mathcal{R} - V(\mathcal{Z})] + \text{H.c.}, \quad (5.1)$$

where we have introduced the covariantly chiral superfield \mathcal{Z} as the Lagrange multiplier. Varying the action (5.1) with respect to \mathcal{Z}^1 gives back the original action (3.2) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})), \quad (5.2)$$

where the function $\mathcal{Z}(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}). \quad (5.3)$$

Eqs. (5.2) and (5.3) define the Legendre transform, and they imply further relations,

$$F'(\mathcal{R}) = \mathcal{Z}(\mathcal{R}) \quad \text{and} \quad F''(\mathcal{R}) = \mathcal{Z}'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))}, \quad (5.4)$$

where $V'' = d^2V/d\mathcal{Z}^2$. The second formula (5.4) is the duality relation between the supergravitational function F and the chiral superpotential V .

6. Modified gravity from supergravity

The field equations (4.3) and (4.4) are easily solved for $R_* = R - iR_{\text{tor}}$,

$$R = \text{Re } R_* = R(X, \bar{X}) = -\text{Re} \frac{9\bar{F} + 3X(4\bar{F}' + 7F')}{F''} - 12\bar{X}X \quad (6.1)$$

and

$$R_{\text{tor}} = R_{\text{tor}}(X, \bar{X}) = \text{Im} \frac{9\bar{F} + 3X(4\bar{F}' + 7F')}{F''}. \quad (6.2)$$

Inverting those functions and substituting the result back into the component action (4.1) gives rise to the modified gravity action (3.3) with

$$\begin{aligned} f(R, R_{\text{tor}}) = F'(\bar{X}) &\left[\frac{1}{3}R_* + 4\bar{X}X \right] \\ &+ 3XF(\bar{X}) + \text{H.c.} \Big|_{X=X(R, R_{\text{tor}})}. \end{aligned} \quad (6.3)$$

7. Weyl transform in components

Let us take $R_{\text{tor}} = 0$ in Eq. (3.3) for even more simplicity, and rescale the function $f(R)$ to $(-1/2\kappa^2)f(R)$ with the gravitational coupling constant κ . Then the action (3.3) is classically equivalent to

$$S_A = \frac{-1}{2\kappa^2} \int d^4x \sqrt{-g} \{ AR - V(A) \} \quad (7.1)$$

where the real scalar $A(x)$ is related to the scalar curvature R by the Legendre transform,

$$R = V'(A) \quad \text{and} \quad f(R) = RA(R) - V(A(R)). \quad (7.2)$$

A Weyl transformation of the metric,

$$g_{\mu\nu}(x) \rightarrow \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right] g_{\mu\nu}(x) \quad (7.3)$$

with an arbitrary parameter $\phi(x)$, yields

$$\begin{aligned} \sqrt{-g}R &\rightarrow \sqrt{-g} \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right] \\ &\times \left\{ R - \sqrt{\frac{6}{-g}} \partial_\mu (g^{\mu\nu} \partial_\nu \phi) \kappa - \kappa^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\}. \end{aligned} \quad (7.4)$$

Hence, when choosing

$$A(\kappa\phi) = \exp\left[\frac{-2\kappa\phi(x)}{\sqrt{6}}\right] \quad (7.5)$$

we can rewrite the Lagrangian in Eq. (7.1), up to a total derivative, to the form

$$\begin{aligned} S_\phi &= \int d^4x \sqrt{-g} \left\{ \frac{-R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right. \\ &\quad \left. + \frac{1}{2\kappa^2} \exp\left[\frac{4\kappa\phi(x)}{\sqrt{6}}\right] V(A(\kappa\phi)) \right\} \end{aligned} \quad (7.6)$$

in terms of the physical (and canonically normalized) scalar field $\phi(x)$.

This procedure is well known in the $f(R)$ gravity theories with *ad hoc* functions $f(R)$ – see e.g. Ref. [11] for a recent review. The $f(R)$ modification of Einstein gravity is the alternative to the dark energy for explaining the present acceleration of the Universe in the IR limit (or large distances).² Our motivation for the $F(\mathcal{R})$ supergravity comes from the UV limit (or small distances) to be described by a UV-complete theory of superstrings, but due to the universal nature of superstrings, the same effective gravity may still be valid in the IR limit (after some renormalization and supersymmetry breaking). Accordingly, we would like to interpret the scalar field ϕ as an *inflaton* in the early Universe and as the *quintessence* in the present Universe, with a scalar potential

¹ Strictly speaking, one should vary the superfield action with respect to an unconstrained pre-potential superfield U defined by $\mathcal{Z} = (\bar{\nabla}^2 - 4R)U$, but it gives rise to the same result.

² It should be $f''(R) > 0$ in order to avoid the so-called Dolgov–Kawasaki instability [14].

$$W(\phi) = -\frac{1}{2\kappa^2} \exp\left[\frac{4\kappa\phi(x)}{\sqrt{6}}\right] V(A(\kappa\phi)). \quad (7.7)$$

It is worth mentioning that the effective gravitational coupling constant κ here may be different from its naive value κ_0 in the IR-limit, due to possible renormalization effects. As regards a space-time torsion in the $f(R)$ -gravity (though unrelated to spin fields), see e.g. Ref. [15].

8. Super-Weyl transform in superspace

A super-Weyl transform of the superfield action (5.1) can be done entirely in superspace, i.e. with manifest local $N = 1$ supersymmetry. In terms of components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal) S -supersymmetry transformation [16]. The chiral density superfield \mathcal{E} is just the chiral compensator of the super-Weyl transformations

$$\mathcal{E} \rightarrow e^{3\kappa\Phi} \mathcal{E}, \quad (8.1)$$

whose parameter Φ is an arbitrary covariantly chiral superfield, $\bar{\nabla}_\alpha \Phi = 0$. Under the transformation (8.1) the covariantly chiral superfield \mathcal{R} transforms as

$$\mathcal{R} \rightarrow e^{-2\kappa\Phi} \left(\mathcal{R} - \frac{1}{4} \bar{\nabla}^2 \right) e^{\kappa\bar{\Phi}}. \quad (8.2)$$

When choosing the super-Weyl chiral superfield parameter to obey

$$\kappa\Phi = \frac{1}{\xi} \ln \mathcal{Z}, \quad (8.3)$$

the super-Weyl transform of the action (5.1) gives rise to the classically equivalent action³

$$S_\Phi = -\frac{3}{\kappa^2} \int d^4x d^2\theta \mathcal{E} \times \left\{ e^{(1+\xi)\kappa\Phi} \left(\mathcal{R} - \frac{1}{4} \bar{\nabla}^2 \right) e^{\kappa\bar{\Phi}} - e^{3\kappa\Phi} V(e^{\xi\kappa\Phi}) \right\} + \text{H.c.} \quad (8.4)$$

or

$$S_\Phi = -\frac{3}{\kappa^2} \int d^4x d^4\theta E^{-1} e^{\kappa(\Phi+\bar{\Phi})} (e^{\xi\kappa\Phi} + e^{\xi\kappa\bar{\Phi}}) + \left[\frac{3}{\kappa^2} \int d^4x d^2\theta \mathcal{E} e^{3\kappa\Phi} V(e^{\xi\kappa\Phi}) + \text{H.c.} \right], \quad (8.5)$$

where we have introduced the full superspace supergravity supervielbein E^{-1} [8].

Eq. (8.5) has the standard form of a chiral superfield action coupled to supergravity, in terms of a Kähler potential $K(\Phi, \bar{\Phi})$ and a chiral superpotential W , with

$$K(\Phi, \bar{\Phi}) = -\frac{3}{\kappa^2} e^{\kappa\Phi+\kappa\bar{\Phi}} (e^{\xi\kappa\Phi} + e^{\xi\kappa\bar{\Phi}}), \quad (8.6)$$

$$W(\Phi) = \frac{3}{\kappa^2} e^{3\kappa\Phi} V(e^{\xi\kappa\Phi}).$$

Therefore, the associated scalar potential is given by the standard formula [17]

$$\mathcal{V}(\phi, \bar{\phi}) = e^K \left\{ \left| \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} W \right|^2 - 3\kappa^2 |W|^2 \right\}, \quad (8.7)$$

where all superfields are restricted to their leading field components, $\Phi| = \phi(x)$.

Eqs. (5.2), (5.3), (8.6) and (8.7) give the algebraic relations between a function F in our supergravity action (3.2) and a scalar potential \mathcal{V} of the classically equivalent scalar-tensor supergravity (8.5). In particular, Eq. (8.7) can be used for embedding inflation into supergravity. Now it can be promoted further, by embedding inflation into the ‘purely geometrical’ modified supergravity (3.2) that determines a Kähler potential and a chiral superpotential of the inflaton superfield in terms of a single holomorphic function F .

9. Discussion

A possibility to achieve inflation by modifying Einstein equations with the 2nd-order curvature terms (representing the gravitational anomalies of the matter fields) was discovered a long time ago [18]. A similar mechanism exist in the four-dimensional supergravity, with inflation generated by the \mathcal{R}^2 -term originating from the one-loop Kähler anomaly [19]. The instabilities in the scenarios based on the 2nd-order curvature terms against adding the higher order scalar curvature terms were discussed in Ref. [20] within perturbation theory.⁴ The inflationary solutions generated by the purely 4th-order terms in the curvatures, in the effective supergravity action generated by superstrings were found in Ref. [21]. Their stability and the scale factor duality invariance were also investigated [21]. In this Letter we emphasize the significance of the full non-perturbative structure of a holomorphic function $F(\mathcal{R})$ (cf. the Born–Infeld-type supergravity [22]). The higher-order curvature terms \mathcal{R}^n are also generated by radiative corrections in supergravity [19] though, unlike superstrings, they cannot be consistent because of the non-renormalizability of supergravity.

In General Relativity, only the spin-2 part of a metric is dynamical. The dynamical generation of a massive scalar field is known to occur already in the presence of the quadratic curvature terms [23], namely, out of the spin-0 part of the metric. In supergravity, as was shown in the preceding section, the whole chiral scalar superfield becomes dynamical, while it can be identified with a super-Weyl compensator – see Eq. (8.3). In superstring theory, the superspin-0 part of the supervielbein is given by a chiral scalar superfield, whose leading complex component represents a *dilaton–axion* field, $\phi| = \varphi(x) + iB(x)$. Hence, we identify $\varphi(x)$ with a superstring dilaton, and B with a superstring B -field (or axion).⁵ As is well known in string theory [6], the dilaton field controls the superstring loops and (D-brane) instantons, which may be the source of the function $F(\mathcal{R})$. The B -field is the source of the non-minimal spacetime torsion in string theory [24].

Unfortunately, the string theory technology at present does not allow us to compute the function $F(\mathcal{R})$ in Eq. (3.2). It is mainly because of the *on-shell* nature of the known string theory that, in principle, can unambiguously fix only the $\mathcal{W}_{\alpha\beta\gamma}$ -dependence of the gravitational effective action [24]. However, its \mathcal{R} -dependence can be fixed by some additional (off-shell) physical requirements such as no-ghosts, stability and the scale-factor self-duality [21], or by going to the IR-limit (weak gravity). For instance, by using chameleon effect, it was demonstrated in Ref. [25] that the function $f(R)$ with

$$f(R) = R + \lambda R_0 \left[\frac{1}{(1 + R^2/R_0^2)^n} - 1 \right] \quad (9.1)$$

with some parameters $R_0 \sim H_0^2$, $\lambda > 0$ and $n > 0$, is fully consistent with *all* Solar System observations. Of course, there are many other

⁴ Those instabilities can be suppressed against the supersymmetric \mathcal{R}^2 terms [19].

⁵ Since the dilaton is already present in the spectrum of superstrings, the coefficient at the kinetic term in Eq. (7.6) should be modified.

³ We have rescaled the action by a factor of $-3/\kappa^2$ and introduced an arbitrary (real) number ξ , while keeping the normalization of Φ arbitrary.

acceptable choices [26], e.g., by the *reconstruction* of the function $f(R)$ from a desired (given) scale factor $a(t)$ via the gravitational equations of motion with

$$R = -6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right]. \quad (9.2)$$

The superfield extension of Eq. (9.2) is given by a superconformally flat superspace with

$$\mathcal{R} = -\frac{1}{4} e^{-2\kappa\Phi} \bar{\nabla}^2 e^{\kappa\bar{\Phi}}, \quad (9.3)$$

so that the function $F(\mathcal{R})$ can also be reconstructed via the equations of motion from a desired history $a(t)$.

References

- [1] A.R. Liddle, D.H. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge Univ. Press, 2000.
- [2] R.H. Brandenberger, *Lect. Notes Phys.* 646 (2004) 127, hep-th/0306071.
- [3] S. Perlmutter, et al., *Supernova Cosmology Project*, *Nature* 391 (1998) 51, astro-ph/9812133; B. Schmidt, et al., *Supernova Search Team*, *Astrophys. J.* 507 (1998) 46, astro-ph/9805200.
- [4] A. Vikhlinin, et al., *Chandra Cluster Cosmology Project*, arXiv:0812.2720 [astro-ph].
- [5] S. Weinberg, *Cosmology*, Oxford Univ. Press, 2008, Section 1.12.
- [6] A. Becker, M. Becker, J.H. Schwarz, *String Theory and M-theory*, Cambridge Univ. Press, 2007.
- [7] M.R. Douglas, S. Kachru, *Rev. Mod. Phys.* 79 (2007) 733, hep-th/0610102.
- [8] S.J. Gates Jr., M.T. Grisaru, M. Roček, W. Siegel, *Superspace or 1001 Lessons in Supersymmetry*, Benjamin–Cummings, 1983; I.L. Buchbinder, S.M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity*, IOP, 1998, Chapter 5.
- [9] T. Biswas, A. Mazumdar, W. Siegel, *JCAP* 0603 (2006) 009, hep-th/0508194.
- [10] S.J. Gates Jr., *Phys. Lett. B* 365 (1996) 132, hep-th/9508153; S.J. Gates Jr., *Nucl. Phys. B* 485 (1997) 145, hep-th/9606109.
- [11] S. Nojiri, S.D. Odintsov, *Int. J. Geom. Methods Mod. Phys.* 4 (2007) 115, hep-th/0601213; T.P. Sotiriou, V. Faraoni, arXiv:0805.1726 [hep-th].
- [12] S. Cecotti, *Phys. Lett. B* 190 (1987) 86.
- [13] T. Kugo, S. Uehara, *Nucl. Phys. B* 222 (1983); T. Kugo, S. Uehara, *Nucl. Phys. B* 226 (1983) 49.
- [14] A.D. Dolgov, M. Kawasaki, *Phys. Lett. B* 573 (2003) 1; A.D. Dolgov, M. Kawasaki, astro-ph/0307442; N.J. Nunes, J.E. Lidsey, astro-ph/0310882.
- [15] S. Capozziello, R. Cianci, C. Stornaiolo, S. Vignolo, arXiv:0810.2519 [hep-th].
- [16] P.S. Howe, R.W. Tucker, *Phys. Lett. B* 80 (1978) 138.
- [17] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, P. van Nieuwenhuizen, *Nucl. Phys. B* 147 (1979) 105.
- [18] A.A. Starobinsky, *Phys. Lett. B* 91 (1980) 99.
- [19] G.L. Cardoso, B.A. Ovrut, *Phys. Lett. B* 298 (1993) 292, hep-th/9210114.
- [20] A.L. Berkin, K. Maeda, *Phys. Lett. B* 245 (1990) 348.
- [21] M. Iihoshi, S.V. Ketov, *Adv. High. Energy Phys.* (2008) 521389, arXiv:0707.3359 [hep-th].
- [22] S.J. Gates Jr., S.V. Ketov, *Class. Quantum Grav.* 17 (2001) 3561, hep-th/0104223.
- [23] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective Actions in Quantum Gravity*, IOP, 1992.
- [24] S.V. Ketov, *Quantum Non-linear Sigma-models*, Springer-Verlag, 2000, Chapter 3.
- [25] A.A. Starobinsky, *JETP Lett.* 86 (2007) 157, arXiv:0706.2041 [astro-ph].
- [26] S. Nojiri, S.D. Odintsov, arXiv:0807.0685 [hep-th].